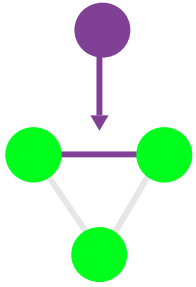
The origins of preferential attachment

Link selection model

Link selection model -- perhaps the simplest example of a local or random mechanism capable of generating preferential attachment.

(a) NEW NODE

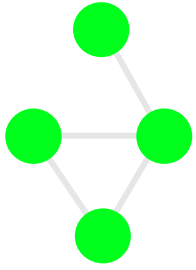


***Growth**: at each time step we add a new node to the network.*

***Link selection**: we select a link at random and connect the new node to one of nodes at the two ends of the selected link.*

To show that this simple mechanism generates linear preferential attachment, we write the probability that the node at the end of a randomly chosen link has degree k as

(b)



$$q_k = Ckp_k$$

In (5.26) C can be calculated using the normalization condition $\Sigma q_k = 1$, obtaining $C=1/\langle k\rangle$. Hence the probability to find a degree-k node at the end of a randomly chosen link is

$$q_k = \frac{kp_k}{\langle k\rangle}, \qquad (5.27)$$

```
#include <stdlib.h>
#include <stdio.h>                    LIBRARIES AND DEFINITIONS
#include <string.h>
#include <math.h>
#include <sys/types.h>
#include <time.h>


#define NodesMax 20000
#define M 2
#define N 1000
#define MATRIX_A 0x9908b0dfUL   /* constant vector a */
#define UPPER_MASK 0x80000000UL /* most significant w-r bits */
#define LOWER_MASK 0x7fffffffUL /* least significant r bits */

static unsigned long mt[N];     /* the array for the state vector  */
static int mti=N+1;             /* mti==N+1 means mt[N] not initialized*/
```

```
main(int argc, char *argv[]) {
  void init_genrand(unsigned long s); /* this is the seed of random
number */
  unsigned long genrand_int32(void);
  double genrand_real2(void), rnddest;
  int i, j, k, edges=0, nodes=N, m=M, m0, degree[NodesMax],
winner[NodesMax], windeg[NodesMax];
  int totdeg, totnds, found, DegDistr[NodesMax], maxdeg=2;
  FILE *fout;
  time_t rawtime;                     DECLARATIONS AND INITIALIZATION
  struct tm *info;

  time(&rawtime);
  info = gmtime(&rawtime);
  j = info->tm_min;
  k = info->tm_sec;
  for (i=1; i<j+k; i++) rnddest = genrand_real2();

  /* open file */
  fout = fopen ("BAedges.txt", "w");
  if (fout==NULL) { printf("no output file\n"); exit(-1); }

  if (argc>2)  sscanf(argv[2], "%d", &m);
  if (argc>1)  sscanf(argv[1], "%d", &nodes);
  if (nodes > NodesMax) {
    printf("Error: too large network, increase NodeMax the header of the
program\n");
    exit(0);
  }
  for(i=m; i<nodes; i++) DegDistr[i]=0;
```

```
m0 = m+m+1;                                          INITIAL GRAPH
for (i=0; i<m0; i++) {
  for (j=i+1; j<m0; j++) {
    edges++;
    fprintf(fout, "%d %d\n", i, j);
  }
  degree[i] = m+m;
}
totdeg = 2*m*m0;
totnds = m0;

/* main loop for creating edges */                    MAIN LOOP
for(i=m0; i<nodes; i++) {
  edges += m;
  degree[i] = m;
  found = 0;
  for(j=0; j<m; j++) {
    rnddest = genrand_real2()*totdeg;
    for(k=0; k<totnds & found<j+1; k++)
    if ((rnddest <= degree[k]) & (degree[k] > 0)) {
      found++;
      windeg[j] = degree[k];
      winner[j] = k;
      fprintf(fout, "%d %d\n", i, k);
      totdeg = totdeg-degree[k];
      degree[k] = 0;
    }
    else rnddest = rnddest-degree[k];
  }
  for(j=0; j<m; j++) {
    degree[winner[j]] = windeg[j]+1;
    totdeg += windeg[j];
  }
  totdeg += (2*m);
  totnds++;
}
```

```
fclose(fout);              CLOSING


/* final reports */
printf("produced %d edges, and %d\n node degree\n",edges,totdeg);
for(i=0; i<nodes; i++) {
  printf(" %5d %5d \n", i, degree[i]);
  if (degree[i]>maxdeg) maxdeg = degree[i];
  DegDistr[degree[i]]++;
}
printf("Graph Degree Distribution\n degree number of nodes\n");
for (i=0; i<=maxdeg; i++) if (DegDistr[i]>0)
     printf(" %5d    %5d\n", i, DegDistr[i]);
}
```

```
/*--------------------- Additional funtions -------------------------*/
void init_genrand(unsigned long s) {
```

RANDOM NUMBER GENERATOR

```
  mt[0]= s & 0xffffffffUL;
  for (mti=1; mti<N; mti++) {
    mt[mti] = (1812433253UL * (mt[mti-1] ^ (mt[mti-1] >> 30)) + mti);
    mt[mti] &= 0xffffffffUL;
  }
}

unsigned long genrand_int32(void){
  unsigned long y;
  static unsigned long mag01[2]={0x0UL, MATRIX_A};
  if (mti >= N) {
    int kk;

    if (mti == N+1) init_genrand(5489UL);

    for (kk=0;kk<N-M;kk++) {
      y = (mt[kk]&UPPER_MASK)|(mt[kk+1]&LOWER_MASK);
      mt[kk] = mt[kk+M] ^ (y >> 1) ^ mag01[y & 0x1UL];
    }
    for (;kk<N-1;kk++) {
      y = (mt[kk]&UPPER_MASK)|(mt[kk+1]&LOWER_MASK);
      mt[kk] = mt[kk+(M-N)] ^ (y >> 1) ^ mag01[y & 0x1UL];
    }
    y = (mt[N-1]&UPPER_MASK)|(mt[0]&LOWER_MASK);
    mt[N-1] = mt[M-1] ^ (y >> 1) ^ mag01[y & 0x1UL];
    mti = 0;
  }

  y = mt[mti++];
  y ^= (y >> 11);
  y ^= (y << 7) & 0x9d2c5680UL;
  y ^= (y << 15) & 0xefc60000UL;
  y ^= (y >> 18);

  return y;
}

double genrand_real2(void) {
    return genrand_int32()*(1.0/4294967296.0);
}
```